SYMAN: a SYMmetry ANalyzer

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Abstract

Symmetry is pervasive in both man-made objects and nature. Since symmetries project to skew symmetries, finding axes of skew symmetry is an important vision task. This paper describes the construction of a symmetry analyzer. Examples using SYMAN on both real and synthetic images are shown. We motivate SYMAN's combination of both global and local methods. We show how to derive a global analytic solution for the skew axes when the degree of skew symmetry is known. We also present a new local tangent-based algorithm which has advantages over previous methods.

1 Introduction

A reflective symmetric planar figure projects, under orthography and weak perspective, to a skew symmetric contour. *Skew symmetry* is defined in [4] as a symmetry at a fixed angle to the axis. An intensity image, with detected edges overlayed, is shown in figure 1 (left). The right side of figure 1 displays SYMAN's recovered axes from the noisy hand-segmented contour shown.

Previous work on finding axes of skew symmetry can be divided into two classes: local and global. Local techniques compare pairs of points. Such points, if they are pairwise skew symmetric, satisfy necessary constraints [3], [6], [7]. Local methods can be robust under occlusion but, because of the number of pairs, are computationally expensive. For unoccluded contours, global moment-based techniques can be more efficient. A moment-based algorithm for finding axes of skew symmetry was proposed in [2] but requires searching two 1D spaces using a heuristic symmetry evaluator. A hough-based method is given in [8] but no results are presented.

The next section presents some examples of the SYMAN's performance. In section 3 we show how to derive an analytic solution for the axes of skewed symmetry, given the number of solutions. In section 4, we present a local tangent-based technique and discuss its advantages. Section 5 discusses the tradeoffs between local and global techniques for recovering axes of skew symmetry. The paper ends with some conclusions and a very brief discussion of future work.

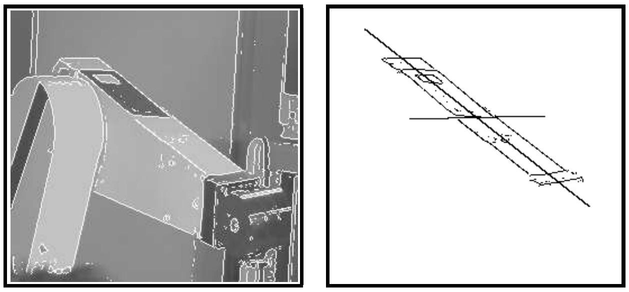

Figure 1: PUMA intensity image with overlayed edges (left) and axes of skew symmetry recovered by SYMAN for the hand-segmented contour shown (right).

2 SYMAN Examples

We now present a few examples of SYMAN's capabilities, some of which are discussed elsewhere in the paper. For more details on SYMAN's capabilities see [3]. Figure 2 is a die image with axes of skew symmetry recovered by SYMAN using both local and global methods. A small camera-imaged contour of a figure from [5] is shown in figure 3, on its right is the deskewed, derotated form recovered by SYMAN. A leaf image, with overlayed edges, is shown in figure 4. The right side of that figure shows the derotated, deskewed contour recovered by SYMAN.

3 Deriving an Analytic Solution for Axes of Skew Symmetry

Given a contour C assume its degree of skew symmetry, $deg(C)$ = number of valid skew symmetry axes, can be determined *á priori*, see [3]. Knowing $deg(C)$ we want to recover all of the skew axes. Assume, for now, that $deg(C) = 3$. We start by computing the four 3^{rd} order moments of C. As noted in [2], any weak perspective or orthographic projection of a symmetric planar figure S can be represented as a image-plane skew followed by a rotation. Without loss of generality, let S be symmetric about the y-axis. The 3^{rd} order moments of C are functions of the 3^{rd} order moments of S, a rotation parameter α and a skew parameter β. Since S is symmetric, the $x^{odd}y^{j}$ moments are zero. Thus, the four 3^{rd} order moments for contour C are functions of α, β, and two 3^{rd} order moments of S. Using elimination theory on the four equations derived for the 3^{rd} order moments

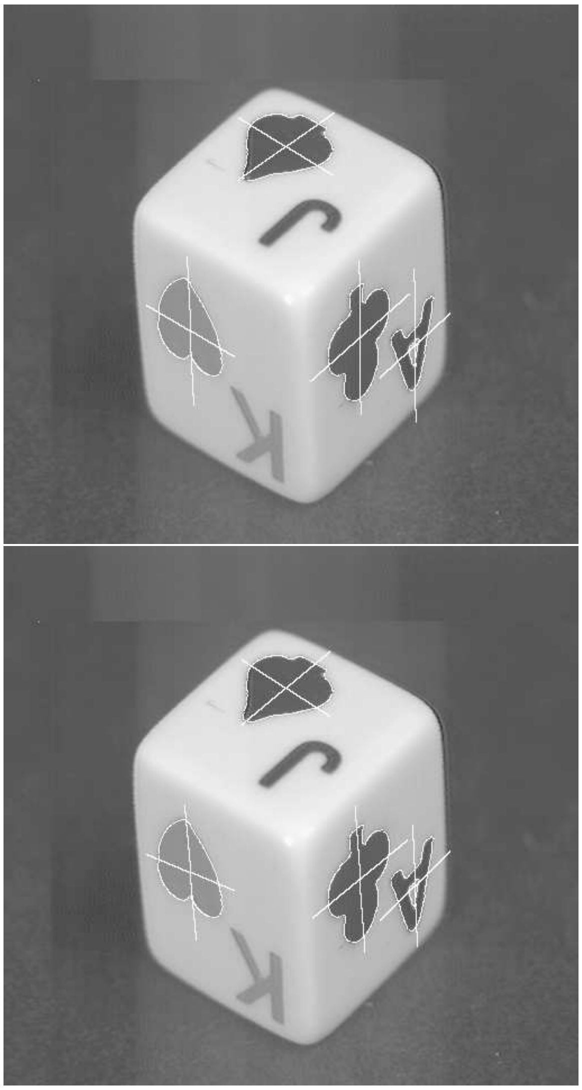

Figure 2: Die image due to Glenn Healey via Jean Ponce. Results using local tangent-based method (top) and moment-based method (botom). Runtimes per contour were 30-60 sec. and 1 sec., respectively.

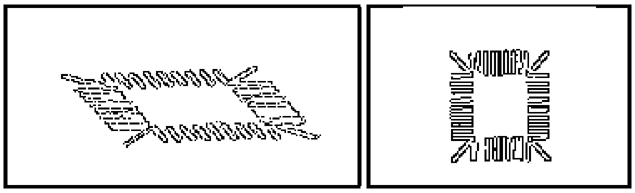

Figure 3: On the left is a skewed and rotated edge image (obtained pointing a camera at a figure in [5]) and on the right is a solution recovered by SYMAN.

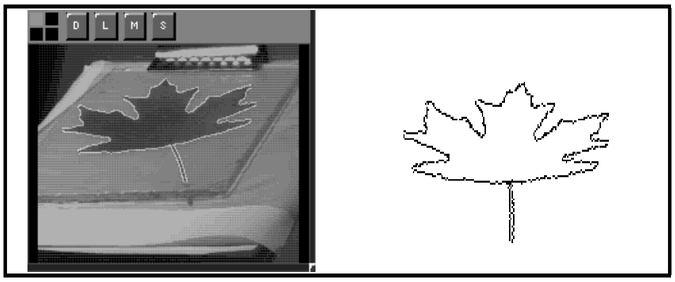

Figure 4: Intensity image of a leaf, with hand-segmented edges of the leaf overlayed (left) and the derotated, deskewed leaf contour recovered by SYMAN.

of C, we can derive a resultant cubic univariate polynomial in rotation parameter α. For each α solution, a corresponding β solution can be obtained.

For any degree of skew symmetry $n = deg(C)$, an analogous procedure is used that relies on the n^{th} order moments of C and S. The n^{th} order polynomial can be precomputed using elimination theory [1]. If we know $deg(C) = n$, the n^{th} order contour moments of C are computed, and the associated n^{th}order polynomial equation in α is solved. SYMAN has the precomputed form of these polynomials through degree 10.

Figure 3 shows a skew symmetric contour c_M, where the original symmetric figure is from [5]. SYMAN hypothesized that the contour c_M is 2-fold symmetric (see [3]), that $deg(c_M) = 4$, and then solved for the 4 sets of axes using the above method. One of the 4 recovered solutions, displayed as a deskewed and derotated contour, is shown in figures 3. When presented with the contour of a skewed and rotated 9-gon and given $deg(c_9) = 9$, SYMAN used the moment-based method to approximately recover all 9 solutions (see [3]).[*]

4 Our Local Tangent-Based Method

Several local methods for finding axes of skew symmetry have been proposed in the literature. These are essentially algorithms constructed around a constraint between skew symmetric pairs of contour points. We propose an algorithm based on a new skew symmetry constraint and contrast it with two existing local methods [6],[7]. SYMAN has running versions of all three methods.

Theorem 1 *Skew Symmetry Tangent Theorem: If a pair of points P_+ and P_- are skew symmetric, then they satisfy a pairwise constraint given by $2 \cdot \cot\beta = \cot\theta_- + \cot\theta_+$, where β is the angle of skew and θ_+ (θ_-) the angle between the transverse axis and the tangent at P_+ (respectively P_-).*

The local tangent-based algorithm considers every pair of contour points, letting each pair vote for a set of

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[*] While the method uses high order moments, something about the ratio of moments that it uses appears to make it reasonably robust. A formal investigation is underway.

Figure 5: Plot of midpoints of pairs (using approach of St. Marc and Medioni) from die's A. Even a robust line fitting process will have difficulties.

α, β values, which are then put into discretized bins. The α value is the slope of the line connecting the points, while β is determined by theorem 1. The discretized α value having the most consistent β value, i.e., smallest standard deviation modulo outliers, is selected.

Consider the contour for the skewed "A", shown in lower right of the left image of figure 2. As shown in that figure, SYMAN was able to recover the axes of skew symmetry using our tangent-based method. As described below, SYMAN's implementation of both the curvature-based algorithm (ala [6]) and the line-based method (ala [7]) failed on this contour.

The curvature-based constraint given in [6] can be written as $-k_+ \sin^3\theta_- = k_- \sin^3\theta_+ k$, where k_+ and k_- are the curvature values at corresponding contour points P_+ and P_-. Since the contours of the letter A in figure 2 are essentially linear, the curvature constraint is satisfied by *almost every* pair of contour points, i.e., $k_+ = k_- = 0$. Thus, this constraint is not useful in recovering the correct set of skew symmetric axes.

The point-based method essentially finds the rotational angle that, when skew pairs are taken along the direction of rotation, has the best-fitting line through the midpoints of the pairs. The point-based method does not require derivative information, but does not reject any pairs either. As a result, the method has localization problems because the ensuing line-fitting routine gets a very noisy point set. This is easily seen in figure 4, a scatter plot of the midpoints for the point pairs from the A contour of figure 2, computed given the approximately correct rotational angle of $\alpha = 43°$. The correct symmetry axis is approximately vertical.

5 Local versus Global Methods

Global techniques for finding axes of skew symmetry consider the entire contour and try to find a solution that derotates and deskews the contour into a pure symmetry. SYMAN's global method uses contour moments to directly recover the axes. The inherent computational complexity is $O(n)$, where n is the number of contour points. Local methods, as in [3],[6],[7], compare pairs of contour points and have an inherent computational complexity of $O(n^2)$, although various techniques (e.g., projections) can be used to reduce the "average" complexity.

To illustrate the speed differences between the 2 methods, we give timings using a Sun SPARCstation1. The methods are applied to the contours in the die image shown in 2. The left figure shows the superimposed axes recovered using SYMAN's local tangent-based technique (c.f., Sec. 4) using the set of hand-segmented edges shown. The times ranged from 30-60 seconds per contour, about the same as those reported in [6]. The axes computed using SYMAN's global moment-based method (c.f. Sec. 3) are qualitatively similar, and appear on the right of Figure 2. The moment-based method's running times were under 1 sec. per contour, a very significant improvement! Having made a (hopefully convincing) argument for the global method on the basis of computational efficiency, the argument for incorporating local methods into any robust skew symmetry analyzer is a simple one: moment-based techniques are sensitive to occlusion while local methods are not (see [3]).

6 Conclusions and Future Work

This paper has outlined some of the aspects involved in designing SYMAN, a symmetry analyzer constructed by the authors. Future work will focus on several areas including canonical forms of skew symmetric contours and detection of skew symmetric regions directly from intensity images.

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