ERROR CHARACTERIZATION OF DETECTION AND MORPHOLOGICAL FILTERING *

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Abstract. Detection followed by morphological and non-linear processing is a typical operator sequence used in machine vision systems. Performance characterization of the sequence involves the derivation of the output image statistics as a function of the input image distribution and the operator tuning parameters. Such a characterization is critical to the task of automating the choice of tuning parameters in various applications. In this paper, we show how one can propagate the error in the detection stage specified in the form of probability of mis-detection and probability of false alarm through morphological operators. The detector output is viewed as a binary random series that is transformed to another binary random series by the morphological operation. An analytical derivation of the relationship between the input and output binary series is rather difficult to characterize. The main essence of the paper is the illustration that the segment and gap length statistics of the output binary series can be numerically computed for morphological filters by using embeddable Markov chains. Theoretical results are verified through simulations. Extensions of the theoretical analysis to handle 2-dimensional filters are also considered in the paper. It is shown that the theory can be modified to characterize 2D morphological filter outputs when the 2D structuring element is decomposable into 1D elements.

Key words: Statistical Characterization, Mathematical Morphology, Embeddable Markov Chains, Binary Random Series

1. Introduction

Pixel neighborhood level feature detection followed by a region level grouping and/or morphological filtering (see [2], [9], [12], [14], [18]) is a typical operation sequence in video/image analysis systems for surveillance and monitoring, document image analysis, machine inspection, etc. The robustness of these algorithms is often questioned because of its use of arbitrary tuning constants that are set by trials and errors. There has been limited research in performance characterization of these algorithm sequences which is critical to the task of automating the choice of tuning parameters in various applications. We view the characterization of the algorithm sequence as the derivation of the

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output image statistics in the grouping step as a function of the statistics of the output image in detection step, the grouping algorithm and its tuning parameters. The difficulty is in defining the statistical models for the input data and the corresponding derivation of the output statistical models. Although there exists prior work in the literature, this problem is difficult to solve even for 1-dimensional signals. In this paper, the output of the detector is viewed as a binary random series. The grouping algorithm output is viewed as another binary random series whose segment and gap length statistics are functions of the probability of false alarm and probability of mis-detection at the detection step. The theoretical derivation of the expressions for the output binary series statistics is rather difficult. However, we use a result from the statistics literature that allows calculation of probabilities of occurrences of run events in a given sequence of Bernoulli trials by embedding the integer random variable into a finite Markov chain. The distribution of the random variable of interest is expressed in terms of the transition probabilities of the Markov chain and can be calculated numerically. We show how the output statistics from operators on binary series (such as morphological openings, closings, etc.) can be derived using the embeddable Markov chain approach. Theoretical results are compared with simulations to validate the correctness of the theory. Extensions of the theoretical analysis to handle 2-dimensional image non-linear filters are also considered. It is shown that the theory can be modified to characterize 2D morphological filtering operation outputs when the 2D structuring element is decomposable into the application of multiple 1D operators.

The paper is organized as follows. Section 2 reviews past work on statistical characterization of vision algorithms and identifies the work closest in spirit to the current work. Section 3 identifies the problem and introduces the embeddable Markov chain approach. Section 4 shows the procedures for computing the statistics of the binary series which is described in terms of segment/gap run length distributions. We show the derivation of the distributions for the output runs obtained through morphological closing (opening) operators for 1D morphological filters in section 5. Section 6 describes how the 1D solution can be extended to handle 2D morphological filters decomposable into a sequence of 1D filtering operations. Section 7 discusses the experiments (simulations) conducted to validate the theoretical results. We present our conclusion and open research issues in section 8.

2. Related Work

There has been work in the computer vision literature on the statistical characterization of vision algorithms. This work has addressed methodological issues and has demonstrated performance analysis of components and systems ([1], [3], [16], [21]). Here the emphasis was on systems analysis, i.e. given a system configuration, the system performance is characterized as a function of tuning constants, ideal image model and perturbation model parameters.

More recent work involves statistical characterization of video analysis algorithms for background adaption [5] and people detection and zooming [7].

In both papers, the change detection step was characterized by the probability of false alarm and the probability of mis-detection. Gao et. al., [5], analyzed background adaption to identify how to formally setup the adaptation parameters to achieve a given error rate, while Greiffenhagen et. al., [7], analyzed total system performance for a people detection and zooming task and automated the selection of the camera control parameters for a pan-tilt camera based on statistical analysis.

Stevenson and Arce [19] were the first to analyze the statistical properties of grayscale morphological filters under Gaussian noise. Costa and Haralick, [10], analyzed the empirical distribution of the output gray levels for gray scale morphological filters. Haralick et. al., [11], described the statistical characterization for morphological filtering operations such as closings and openings on input 1D binary signals, however, their input signal model was rather restrictive. The closest related work involves performance characterization of boundary segmentation methods [17]. The paper provides a theoretical analysis of edge detection, linking and grouping processes and provides a model for boundary fragmentation of a given ideal boundary due to random noise and the edge strength threshold. In addition that paper describe the output statistics of the alternating segment and gap process after a morphological closing operation (i.e. a gap filling operation). It is also worth noting that the analyses presented in [17] are continuous domain analyses. They are limited in the sense that they derive the fragmentation process statistics for a given probability of detection uniform over the entire boundary. Our paper addresses a more general model and allows for non-stationarity in the input process.

3. Embeddable Markov Chain Approach

This section considers the problem of detection and grouping illustrated in figure 1. The main problem is to relate the statistics of the binary series in the output to that of the binary series in the input, the morphology operator and its parameters.
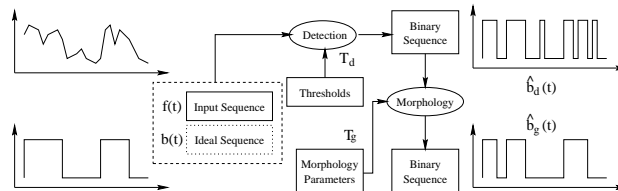
3.1. PROBLEM STATEMENT

Fig. 1. System Diagram

More formally, let $f(t)$ denote the mapping from the pixel index set $\{1, 2, \ldots, N\}$ to the gray level measurements R. Let $b(t)$ denote the ideal unknown function representing the mapping that assigns the true labels (e.g. foreground/background

(1/0)) for each index. The detection algorithm is viewed as a function that maps $f(t)$ to the binary series $\hat{b}_d(t)$ by using a decision rule (that could be spatially varying). Define $p_f(t)$ as the conditional probability that the detector output $\hat{b}_d(t) = 1$ given that the true label $b(t) = 0$. Let $p_m(t)$ be the conditional probability that $\hat{b}_d(t) = 0$ given $b(t) = 1$.

A natural representation of the statistics of $\hat{b}_d$ and $\hat{b}_g$ is the distribution of the run lengths as a function of the filter parameters used (e.g. structuring element sizes in a morphological operator). This representation is convenient in the fact that it provides a natural way to interpret the results of morphological operations. The size distributions of shapes (granulometries) have been used in the morphology literature to describe signal statistics (e.g. [2], [14], [18]). We therefore pursue the derivation of the statistics of the run lengths as a function of $b(t)$, $\hat{b}_g(t)$, and the morphological operator parameter T_g. Let $\hat{C}_l$ denote the number of runs of length l in the output of the morphological algorithm. Our objective is to derive the conditional distribution $p(\hat{C}_l; T_g, N | \{p_m(t)\}, \{p_f(t)\})$. Most research in distribution theory of runs, [13] [15], has addressed this problem by using combinatorial analysis. However, past work assumed stationarity (i.e. $p_m(t) = p_m$ and $p_f(t) = p_f$), while we do not make any assumptions about the form of $p_m(t)$ and $p_f(t)$.

Our approach is to use a technique developed in [4] that embeds a discrete random variable into a finite Markov chain to numerically compute the probability mass function (pmf) of the discrete random variable. The pmf is essentially computed as a function of the N-step transition probabilities of the Embeddable Markov Chain (EMC). The main advantage of using the algorithm is that Monte-Carlo simulations are prohibitively slow when probabilities for unlikely events are being estimated.

In the next section, we proceed with a formal description of an EMC and how it's useful to compute the pmf for a discrete random variable. For more details, please see the original paper, [4].

3.2. EMBEDDABLE MARKOV CHAIN APPROACH

For a given n, let $\Gamma_n = \{0, 1, \cdots, n\}$ be an index set and $\Omega = \{a_1, \cdots, a_m\}$ be a finite state space.

A nonnegative integer random variable X_n can be embedded into a finite Markov chain if:

1. there exists a finite Markov chain $\{Y_t : t \in \Gamma_n\}$ defined on the finite state space Ω.

2. there exists a finite partition $\{C_x, x = 0, 1, \cdots, l\}$ on the state space Ω.

3. for every $x = 0, 1, \cdots, l$, we have $p(X_n = x) = p(Y_n \in C_x)$.

Let Λ_t be the $m \times m$ transition probability matrix of the finite Markov chain $(\{Y_t : t \in \Gamma_n\}, \Omega)$. Let U_r be a $1 \times m$ unit vector having 1 at the rth coordinate and 0 elsewhere, and let U_r' be the transpose of U_r. Finally, for every C_x, define the $1 \times m$ vector $U(C_x) = \sum_{r:a_r \in C_x} U_r$.

If X_n can be embedded into a finite Markov chain, then $p(X_n = x) =$

$\pi_0 \left(\prod_{t=1}^{n} \Lambda_t \right) U'(C_x)$ where $\pi_0 = [p(Y_0 = a_1), \cdots, p(Y_0 = a_m)]$ is the initial probability of the Markov chain.

If the Markov chain is homogeneous (i.i.d. case), that is $\Lambda_t = \Lambda$ for all $t \in \Gamma_n$, then $\forall x = 0, \cdots, l$ the exact distribution of the random variable X_n can be expressed by $p(X_n = x) = \pi_0 \Lambda^n U'(C_x)$.

In order to find the distribution for any embeddable random variable, one has to construct: (i) a proper state space Ω. (ii) a proper partition $\{C_x\}$ for the state space, and (iii) the transition probability matrix Λ_t associated with the EMC. The exact process by which the state space is defined along with the partitioning is dependent on the nature of the statistic of interest and the operator used.

4. Statistics Calculation by using EMC Approach

Before we address the problem of deriving the run length distributions in the output of a morphological algorithm, we first show how the EMC approach can be used to derive the run length distribution of the observation of an uncorrelated random binary series. Here, we wish to address the problem of calculation of the joint run length distribution, i.e. "What is the probability of having m runs with size M and n runs with size N?"
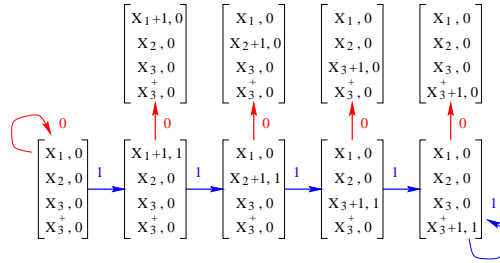

Fig. 2.   Diagram of Run Length Statistics Calculation Example

State Space Construction: The state space construction for the computation of the distribution of run lengths is rather straightforward. View the sequence of binary observations up to pixel T as partial observations of the 0 and 1 runs. We need a variable x_i to denote the number of observations of runs of a given length i at pixel T and an indicator variable m_i to denote the situation whether the preceding number of ones is exactly equal to i or not. Thus m_i takes on value 1 if the last sequence of 1s is exactly equal of length i and 0 otherwise. The pair $(\mathbf{X}, \mathbf{M})$, $\mathbf{X} = [\mathbf{x_1}, \dots, \mathbf{x_n}, \mathbf{x_n^+}]$, $\mathbf{M} = [\mathbf{m_1}, \dots, \mathbf{m_n}, \mathbf{m_n^+}]$ denotes the states for the problem. Here x_n^+ denotes the number of runs larger than n and m_n^+ is the corresponding indicator variable. Given these states it is easy to see that the graph shown in figure 2 constitutes the Markov chain for the run length statistics computation problem. In the graph, we focus on the joint distribution of run length whose size is equal to or less than 3. We

can simply extend the graph to meet the requirement of the joint distribution of longer run length.

State Space Partition and Definition for Λ_t**:** The partition of the state space corresponds to the singleton sets of **X** with assigned count values. The values for the probabilities in Λ_t are given by the following expressions. For example, the probability of observing a 0 at location t is given by the sum of two terms: the probability that the true value is 0 and there is no false alarm, and the probability that the true value is 1 and there is a misdetection.

$$q_t(0) = p_{b(t)}\{0\}(1 - p_f(t)) + p_{b(t)}\{1\}pm(t) \tag{1}$$

$$q_t(1) = p_{b(t)}\{0\}p_f(t) + p_{b(t)}\{1\}(1 - pm(t)) \tag{2}$$

where $p_{b(t)}\{\}$ is the distribution of the ground truth.

It is clear that a large state space is needed for calculating the joint distribution when n is large. For example, when $n = 50$, more than 10^{24} states are needed, thus needing large memory for implementation. An incremental approach to handle this problem is currently being investigated.

In image analysis problems one is not necessarily interested in the event of observing the number of runs of a given length in a finite interval. Rather, the interest is in obtaining the ratio of the number of runs of a given length to the total number of runs in the interval. We use the term "weight distribution", [6], to describe the distribution of $\{\frac{N_n}{N}\}_{n=1}^N$, where N_n is the number of runs with run length size n and N is the total number of runs in the interval.

5. Statistics for One Dimensional Morphology

In the previous section we have shown how we can derive the run-length statistics for a binary random series before the application of the morphological algorithm. The statistics can be derived for an uncorrelated binary series or a correlated series defined in terms of a homogeneous or inhomogeneous Markov chain [6]. In this section, we use the probability of observing a given number of runs of length greater than or equal to S, $G_{n,S}$, after the closing operation with closing parameter $T_g = L$ as an example to illustrate how the output statistics of a morphological operator for binary series can be derived by using the EMCs. The trick again is to devise the appropriate EMC. Similar EMCs can be devised for openings, and openings followed by closings, etc.

State Space Construction: To construct the state space we have to consider the property of the closing operation. Closing essentially fills gaps of sizes less than a given length L. At any given pixel the output $\hat{b}_g(t)$ is a 1 if and only if $\hat{b}_d(t) = 1$ or $\hat{b}_d(t) = 0$ and there exists two neighbors with indices $t - i$ and $t - j$, $i, j \geq 1$, $\hat{b}_d(t - i) = 1$ and $\hat{b}_d(t + j) = 1$ with $j + i < L$. This implies that in addition to the number of runs of length greater than or equal to S, the state space has to include information about the run length of the last 1-run observed as well as the length of the last gap (if any) (0-runs) to identify the partial state. One has to wait until the gap length is greater that L before deciding to terminate a previous run.

Formally, define the state variable to be (x, s, f) where $x = 0, 1, \ldots, [\frac{n+1}{k+1}]$ denotes the number of success runs of size greater than or equal to k, $s = -1, 0, \ldots, L$ is the number of successes in the last success run and $f = -1, 0, \ldots, L$ is the number of failures in the last failure run, L is the size of the structuring element. The value -1 for f corresponds to a gap of length greater than L that cannot be filled, while a value -1 for s corresponds to having a 1-run of length greater than or equal to S. The left graph of figure 3 corresponds to the initial condition for the state transitions. The right graph of figure 3 corresponds to the state transition diagram illustrating that the length of 1-run observed before the start of the transitions is already greater than or equal to S (i.e. the overflow condition). Figure 4 corresponds to the elements of the state transition diagram for the case that the partial 1-runs observed have length $k < S$ and $k + L + 1 < S$. Figure 5 corresponds to the elements of the state transition diagram for the case that the partial 1-runs are such that the constraint $k + L + 1 \geq S$ is satisfied. Note that the diagrams are illustrative of only the portions of the large state transition diagram for the Embeddable Markov Chain. For illustration purposes we present only the parts of the diagram that are the elements of the bigger graph. The bigger graph is the concatenation of these individual elements over all (x, s, f) values.

State Space Partition and Definition for Λ_t**:** The partition of the state space is based on the number of success runs, x. The values of transition probabilities are given by q_t and $1 - q_t$ where

$$q_t = p_{b(t)}\{0\}p_f(t) + p_{b(t)}\{1\}(1 - p_m(t)) \tag{3}$$
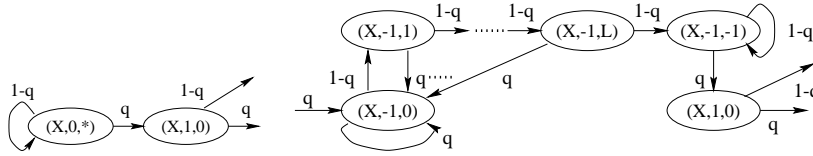

Fig. 3. State Transition for Closing Operator, Initial condition for Success Runs (Left); Success Run Length Overflow (Right)
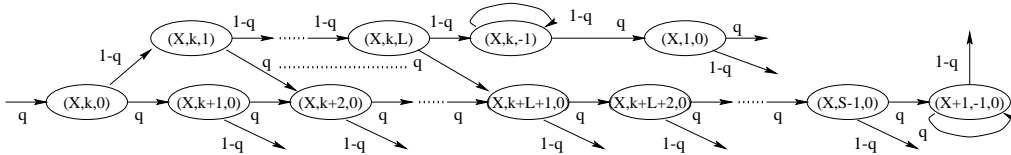

Fig. 4. State Transition for Closing Operator, Small Lengths

The statistics of runs for close-open or open-close operations can be computed in a similar fashion as for the closing. The main difficulty is in the definition of the transition diagrams for the Markov chain. The graph is more
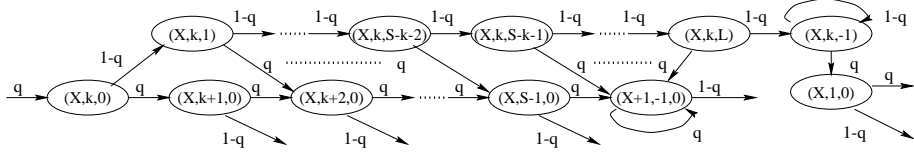
Fig. 5.   State Transition of Closing Operator, Large Success Run Lengths

complex, but not infeasible to construct. The main idea is to try to use more state variables to save all the temporary information and extend the graph from a single layer representation to multi-layered graph.

6. Statistics for Morphology in the 2D Case

To our knowledge the literature for the two dimensional case focuses on either the coverage processes, [8], spatial point processes and random sets, [20]. In this section, we provide a method that allows us to do some analysis when the 2D operations could be decomposed into the sequential application of two 1D structuring elements, horizontal and vertical for instance, [9].

Assume that we can decompose the two dimensional operation into a two step process. The first step is easy to analyze since we can just use the previous procedure. However, the second processing step cannot use the results of the 1D analysis because one has to take into account the fact that the data in the direction of the operator application is correlated[1]. Even if the analysis assumes that the binary series is independent, the independence assumption will no longer hold in the morphological algorithm output for features in that direction. However, if two filters are applied in orthogonal directions, say vertical then horizontal, after the vertical pass the horizontal features is still satisfy the independence assumption. Thus we can use the 1D analysis for analyzing the statistics of the runs in the horizontal direction. The only thing we have to do is to recalculate the new $p_f(t)$ and $p_m(t)$ since these have been altered as a result of application of the morphological operation in the vertical direction. That is, a given pixel with low value of $p_f(t)$ will have a higher value of $p_f(t)$ if a vertical closing were applied.

We will use the closing operator as an example to explain the details. Suppose the closing parameter is L. Let p_{SL} be the probability that, at each side of t, there exists at least one pixel that is a success and the distance between the success pixels on both sides is less than L. According to the closing rules,, the pixel t will have a success value when the distance is less than L.

$$p_f^{new}(t) = p_f^{old}(t) + (1 - p_f^{old}(t)) \cdot p_{SL} \qquad (4)$$

$$p_m^{new}(t) = 1.0 - [P_{SL} \cdot p_m^{old}(t) + (1 - p_m^{old}(t))] \qquad (5)$$

For example, suppose we have a sequence $a'b'tab$ and the closing parameter is

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[1] We have used the EMC approach to evaluate distributions for multiple close-open operations applied in sequence. Details will be presented in a subsequent paper.

2. What is the probability of p_{SL}? Let $p_{FL} = 1 - p_{SL}$, then,

$$p_{FL} = p(a = F, b = F) + p(a = F, b = S, b' = F) + p(a = S, a' = F, b' = F) \qquad (6)$$

In general the methods described above cannot be applied to several morphological operations applied in sequence. If one calculates the weight distance before and after the closing operator, one can see that spatial correlation between pixels directly affects the distribution. Even though they are independent before the morphology operator, the pixels are correlated after the operator. Usually, after reestimating the probabilities of miss detection and false alarm, we still cannot apply another morphology operator without validating the independence assumption, or by coming up with a Markov Chain approximation to the correlated series. The problem is that the order of this chain changes as a function of the structuring element parameters used. Our calculations for 2D morphology works only because of the independence assumption. In general, as long as the independent assumption is validated in the original data and the two directions are orthogonal, after applying a 1D morphology operator in one direction, we can apply another 1D morphology operator in the other direction by reestimating the false alarm probability and miss detection probability. The method can be extended to 3D orthogonal morphology operations.

7. Simulation Validation

As in previous work on analysis of Morphology we present simulation results to validate the theoretical results. It can be applied on real data, if one verifies that the input assumptions hold, but the acquisition of ground truth for real data is quite problematic.
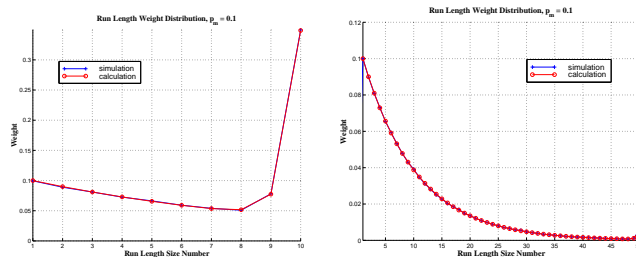

Fig. 6.   Run Length Weight Distribution, 1D

Figure 6 shows the simulation and theoretical calculation results of run length weight distribution with sequence length 10 and 50. The variance of the weight distribution (not shown in the figure) is larger comparing to the value of the mean than usual. This can be explained as follows: In the simulation, we fix the total number of pixels to be M. Then, the total number of runs in the sequence is also a random variable. For any given number of runs, N, the

estimate of the weight is $\hat{q}_u | N \sim N(q_u, \frac{q_u(1-q_u)}{N})$.

$$E\{\hat{q}_u\} = q_u \tag{7}$$

$$\sigma^2[\hat{q}_u] = q_u(1-q_u)\sum_N \frac{1}{N}p(N) \tag{8}$$

Figure 7 shows the simulation and theoretical calculation of the run length weight distribution for 2D closing operation. In the experiment, independence assumption is assumed and closing parameter is 2 by 2. The effect of closing operator is clearly shown.
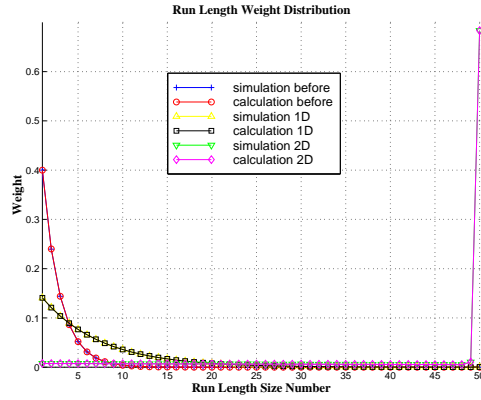

Fig. 7.   Run Length Weight Distribution, 2D

Table I gives the calculation and simulation results for the parameter reestimation for 2D closing case. The original false alarm probability is 0.4.

TABLE I
Probabilities Reestimation

μ	$\hat{\mu}$	$\hat{\sigma}$
0.4	0.400043	0.0157815
0.07168	0.0716085	0.0170479 (H stat.)
	0.0716085	0.00762734 (V stat.)
$9.99e - 5$	0.000118256	0.000669619

Additional experiments not shown here were conducted to validate the theory. We refer the reader to a technical report, [6].

8. Conclusion

In this paper, we introduce the embeddable Markov chain approach for computation of statistics of runs in the output of detection and grouping stages.

The method can also be applied to analyze other non-linear filters. Through simulation, we show that as long as the input model is valid, we can use this approach to compute the statistics rather than doing a brute force simulation to get the statistics. Developing techniques to validate the input model for real data is an area of ongoing research. The results presented herein can be used in any aspect of vision processing using morphology to automate the parameter selection (e.g. structuring element sizes). The verification of this aspect is a subject of ongoing research. Extension of the method for repeated, non-separable 2D operators is an open research issue.

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